PRIMA 2009 Agent School: Mechanism Design

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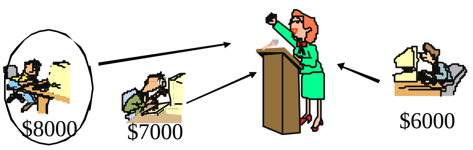
Outline

- Examples
- Basic Terms
- Pareto Efficiency/Prisoners' Dilemma
- Mechanism Design
- Single-item Auctions
- Combinatorial Auctions
- Clarke Tax/Re-distribution

Example: Standard (First-price) Sealed-bid Auction

Protocol: Each bidder (agent) privately bids the price it is willing to pay, and the highest bidder wins; pays the value of its bid.

Demerit: Spying other agents' bids is profitable.

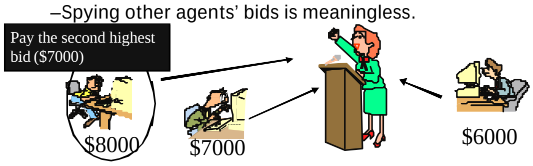


Second-price Sealed-bid Auction (Vickrey Auction)

Protocol : The highest bidder wins, but pays the value of the second highest bid.

Characteristic:

- –Bidding its true evaluation value (the maximal value where it does not want to pay any more) is the optimal strategy (incentive compatibility).
- –Spying other agents' bids is meaningless.



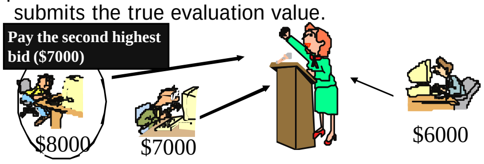
Spying is Useless Because…

Assume its own evaluation value is $8000, and found that others' highest bid is:

(I) less than $8000: its payment is the same to the case when it submits the true evaluation value (without spying).

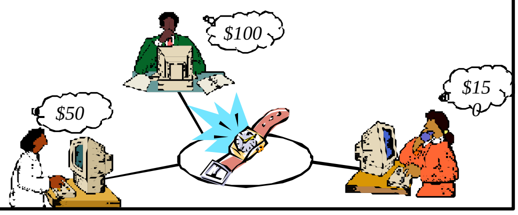
(II) more than $8000: the agent cannot obtain positive utility anyway.

Its profit cannot be more than the case when it truthfully submits the true evaluation value.



Quiz: The larger also serves for the smaller?

- If the highest bidder wins, and pays the value equals to the third highest bid, is honesty still the best policy?



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Sponsored Search

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Sponsored Search

- Each advertiser bids a price for each keyword.
- When the keyword is searched by a user, then the ads with top n bids are shown to the user (with a hyperlink to the advertiser's page).
- The advertiser needs to pay only when the user actually clicks the ad (pay-per-click).

Issue: how much should the advertiser pay?

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Pricing Mechanism

- In early systems, the advertiser pays the amount equals to its bid (first-price)
  - The advertiser keeps on sending dummy search request, checks its position, and adjusts its price.
  - The bid prices keep on changing/unstable.
- In recent systems (including Google), the advertiser, who wins the k-th slot, pays the price equal to the k+1-st bid (second-price).
  - The bid prices become more stable.

The most frequently used auction protocol in the world!

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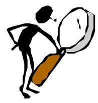
Theory of Auctions

- For seller:
  - The behaviors of bidders change according to the auction protocol.
  - can give a protocol that can achieve socially desirable outcome or robustness against cheating.
- For bidders:
  - The best strategy changes according to the auction protocol.
  - can give a method for finding the best strategy.



important technology for Electronic Commerce

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Characteristics of Player

risk neutral/averse

- risk neutral：only cares the expected utility
  - e.g., indifferent between two lotteries:
    1) he/she obtains 0 for the head and $100 for the tail,
    2) he/she obtains $50 for sure.
- risk averse：prefers that is more certain （even with less expected utility）
  - e.g., prefers getting $45 for sure to 1.

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Assumption for Simplicity

Quasi-linear utility: an agent's utility is defined as the difference between its evaluation value of the allocated good and its payment.

- If an agent wins a good whose evaluation value is $100 by paying $90, its utility is $100-$90=$10.
- If it does not win, its utility is $0.
- If it does win, paying $100, its utility is 0.

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The St. Petersburg Paradox

Assume the following lottery...

- Flip a coin, if it comes up tail, you get $2.
- If head, flip a coin again, if it comes up tail, you get $4.
- If head, flip a coin again, ...
- If it comes up tail the first time at n-th trial, you get $\$2^n$.

How much are you willing to pay to participate this lottery?

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Definitions of Basic Terms

Strategy: the way for choosing actions, i.e., how to decide the bidding price

Dominant Strategy: the best strategy regardless of the actions of other agents

- Paper-rock-scissors: no dominant strategy
- If only paper and rock are allowed, paper is the dominant strategy

Dominant Strategy equilibrium: assume each agent has a dominant strategy. Then, the combination of dominant strategies is called dominant strategy equilibrium.

Nash equilibrium: a weaker concept than a dominant-strategy equilibrium. A combination of strategy is a Nash equilibrium if each strategy is a best response to other strategies.

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Desirable Properties of a Group Decision Making Mechanism

- For each agent, there exists a dominant strategy.
- The mechanism is robust against various frauds (e.g., spying).
- A Pareto efficient outcome can be achieved.

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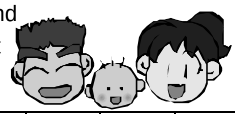
Pareto Efficiency

Definition:

A state is said Pareto efficient, iff there exists no state that is

- better for one agent, and
- no worse for all the rest



×	Movie	2	2	2
	Shopping	2	2	5
	Zoo	2	3	1
×	Home	1	1	1

Pareto Efficiency (cont'd)

- Divide $1000 between you and me.
- We can throw away some.
- (0, 1000), (x, 1000-x), (1000, 0), all are Pareto efficient.
- We can agree on (250+x, 750-x) is better than throw away $500 and get (250, 250).

Pareto Efficiency (cont'd)

- In principle, it is not clear whether we can compare utilities of different people.
  - Do we really have a common measure (e.g., money)?
- The definition of Pareto efficiency can be applied even if we cannot compare utilities.
- We can consider Pareto efficiency is a minimal requirement of a desirable social choice.
- If x is not Pareto efficient, then there is another choice x', where everybody prefers x' to x (or at least the same).

Rational Player

- A rational player tries to maximize its utility by all means, using possibly unlimited computational power.
- In a zero-sum or constant sum game, all outcomes are Pareto efficient.
- If a game is not zero/constant-sum, there is a possibility that both can be happy at the same time (to some extent).
- If players are irrational/incompetent, then both might become unhappy.
- However, if players are rational, then the result should be Pareto efficient.
  - It is unlikely that very clever players throw out some utilities.

Prisoners' Dilemma

- The police arrests two suspects (prisoners).
  - If both of them do not confess, both will be released.
  - If one confesses, while the other does not confess, the one who confessed will get a reward, while the other will get a severe punishment.
  - If both confess, both receives a normal punishment.

I \ II	D	C
D	2, 2	4, 1
C	1, 4	3, 3

Prisoners' Dilemma

- (C, C): (3, 3) is Pareto efficient; both player can agree it is better than (D,D): (2, 2)
- It sounds irrational that a dominant strategy equilibrium is not Pareto efficient!
- Although players are very clever, they cannot reach a Pareto efficient situation.

If this game is repeated...

- Assume the game is repeated three times.
- Let's find an Iterated Dominance Equilibrium
- Let's consider the last (third) game.
- Your rational opponent will deceive you anyway.
- Then, it is meaningless to give a favor at the second game.
- Then, both player will deceive at the second game.
- Similarly, in the first game, both rational player will deceive.
- This is the same even if the game is repeated 1000 times...
- Note: if your opponent is irrational, then you might be better off by cooperating.

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Prisoners' Dilemma Tournament

- Held by Robert Axelrod (Univ. Michigan, political scientist)
- Computer programs repeatedly play Prisoners' Dilemma.
- The program that obtains the highest total score wins.

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Prisoners' Dilemma Tournament

- Surprisingly, a very simple program wins (called Tit-for-Tat)
- Cooperate in the first game.
- Then imitate the opponent's play in the previous round.
- If the opponent deceives, then it retaliate with D.
- As long as the opponent cooperate, it keeps on cooperate.
- Characteristic: Not persistent.

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Prisoners' Dilemma Tournament

- There was a second tournament.
- Many programs try to beat Tit-for-Tat.
- However, Tit-for-Tat won again!
  - Actually, a new program beats Tit-for-Tat.
  - But when two new programs face each other, they often deceive and their obtained scores are low.
  - Tit-for-Tat does not defeat anybody, but plays relatively well for everybody.
  - As a result, the total score was highest.
- Lesson: the goal is not to defeat your opponent, but to receive the high score in total.
  - You need a strategy that can draw cooperation from the opponent.

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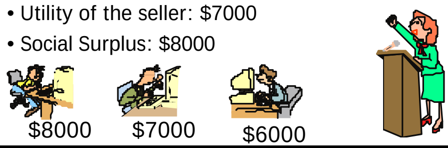
Pareto Efficiency in Auctions

- The social surplus must be maximized; the good must be allocated to the agent who has the highest evaluation value.
  - The agent whose evaluation value is $8000 wins and pays $7000.
  - Utility of this agent: $8000 - $7000=$1000
  - Utility of the seller: $7000
  - Social Surplus: $8000



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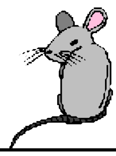
Mechanism Design

- Designing a mechanism is determining the rules of a game.
- The designer cannot control the actions of each agent.
  - No way to force an agent to be honest or to refrain from doing frauds.



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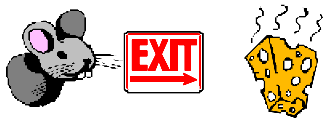
Mechanism Design

How can a designer achieve a certain desirable property (e.g., Pareto efficiency)?

- Design rules so that:
  - For each agent, there exists a dominant strategy.
  - In the dominant strategy equilibrium, the desirable property is achieved.



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Incentive Compatibility

Direct revelation mechanism: directly ask types/evaluation values for each agents

Incentive compatibility: A direct revelation mechanism is (dominant-strategy) incentive compatible if truth-telling is a dominant strategy for each agent.

Revelation Principle: If a certain property (e.g., Pareto efficiency) can be achieved in a dominant strategy equilibrium using an indirect mechanism, that property can be achieved using an incentive compatible direct revelation mechanism.

We can restrict our attention only to (incentive compatible) direct revelation mechanism!

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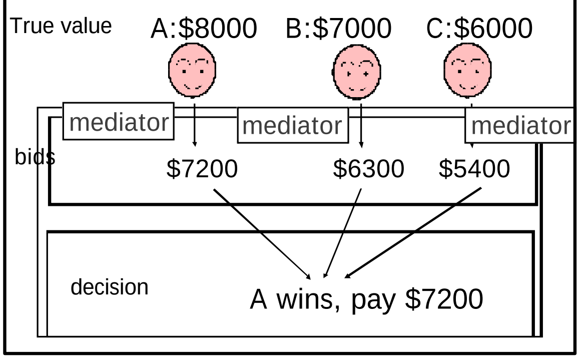
Revelation Principle



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Private/Common/Correlated Values

Private Value: each agent knows its value with certainty, which is independent from other agents' evaluation values (e.g., antiques which are not resold).

Common value：the evaluation values for all agents are the same, but agents do not know the exact value and have different estimated values (e.g., US Treasury bills, mining right of oil fields).

Correlated Value: Something between above two extremes.

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English (open cry)

Protocol：Each agent is free to revise its bid upwards. When nobody wishes to revise its bid further, the highest bidder wins the good and pays its own price.

Dominant strategy (in private value): keep bidding some small amount more than the previous highest bid until the price reaches its evaluation value, then quit.

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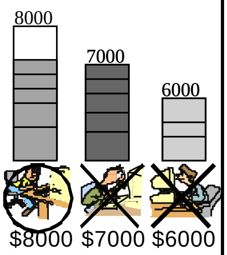
English (open cry)

- In the dominant-strategy equilibrium, the agent with the highest evaluation value wins and pays the second highest evaluation value +e.
- The obtained allocation is Pareto efficient.



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Vickrey (Second-price Sealed-bid)

Protocol：Each agent submits its bid without knowing other agents’ bids. The agent with the highest bid wins and pays the value of the second highest bid.

Dominant strategy (in Private value)：Bidding its true evaluation value is a dominant strategy

- The obtained allocation in a dominant strategy equilibrium is Pareto efficient.
- The obtained result is identical to English in the dominant-strategy equilibrium.

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English Auction and Revelation Principle

- In English Auction, each bidder has a dominant strategy and the allocation is Pareto efficient in a dominant strategy equilibrium.
- Thus, there must be a corresponding direct revelation mechanism that can achieve the same outcome.
  - Proxy bidding: a user specifies the largest amount he/she is willing to pay, then a software/proxy automatically bids.
  - No incentive for lying to the proxy.
  - If we assume the proxy is a part of the mechanism, this mechanism is dominant-strategy incentive compatible and Pareto efficient.

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Common Value Auctions

- English and Vickrey can be different.
  - Agents can obtain more information in English.
  - Can revise the estimation using the obtained information.

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Winner’s Curse

- In a common value auction, each agent does not know the real value of the auctioned good (which is common to all agents).
- Each agent has a different estimated value.
- Unless the agent has an especially good piece of information, the winner tends to be the agent that has the largest estimation error.
- If an agent increases its bid too close to its estimated value, the expected utility can be negative.

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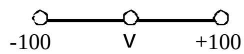
Winner’s Curse (Example)

Settings: there are two bidders, the estimated value of the bidder can be either under-estimated (v-100) or over-estimated (v+100), where v is the real common value. Both are equally probable, i.e., each probability is 1/2.

- In a first-price sealed-bid auction, a bidder might (wrongly) think as follows:

  My estimated value is right in average. Thus, if I bid my estimated value - 40, my expected utility would be +40.



Winner's Curse (Example)

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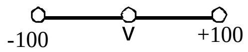
- Assume ties are broken by tossing a coin.
- Assume another bidder uses the same strategy:
  - There are 4 possible combinations of bids.
  - (v+60, v-140): with probability 1/4, the agent wins, the utility is –60.
  - (v+60, v+60): by tossing a coin, with probability 1/8, the agent wins, the utility is -60.
  - (v-140, v-140): by tossing a coin, with probability 1/8, the agent wins, the utility is 140.
- The expected utility is: -40/8 = -5



Quiz: Winner's Curse

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- Assume you are considering buying company A.
- The value of A (i.e., v_A) is uniformly distributed between [0, 100].
- If you buy A, you can increase its value to 50% and sell.
- If you offer b, the owner of A will sell if b > v_A.
- If you buy at price b, your profit is 1.5 v_A — b.
- How much should you offer?

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Combinatorial Auction

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- Multiple different goods with correlated values are auctioned simultaneously.
  - Complementary: PC and memory
  - Substitutable: Dell or Gateway
- By allowing bids on any combinations of goods, the obtained social surplus/revenue of the seller can increase.
- e.g., FCC spectrum right auctions

Vickrey-Clarke-Groves Mechanism (VCG) aka Generalized Vickrey

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- Each agent declares its evaluation values for subsets of goods.
- The goods are allocated so that the social surplus is maximized.
- The payment of agent 1 is equal to the decrease of the social surplus except agent 1, caused by the participation of agent 1.
- Satisfies incentive compatibility and Pareto efficiency.

An Example of the VCG

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Setting: three agents (agent 1, 2, 3) are bidding for two goods.

	coffee	cake	both
~~Agent 1~~	~~$6~~	~~$0~~	~~$6~~
Agent 2	$0	$0	$8
Agent 3	$0	$5	$5

Result:

- Agent 1 gets the coffee, 3 gets the cake.
- Agent 1 pays $8－$5＝$3.
- Agent 3 pays $8－$6＝$2.

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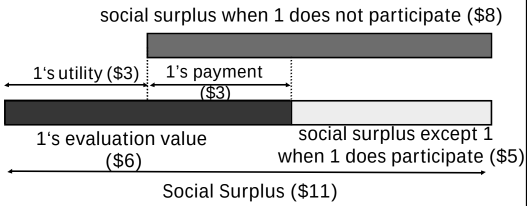
Incentive Compatibility of VCG

- Goods are allocated so that social surplus is maximized.
- An agent can maximize its utility when the social surplus is maximized.



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Quiz: Clarke Tax

- The VCG is one instance of the Clarke mechanism (a.k.a. Vickrey-Clarke-Groves mechanism, Clarke Tax).
- Can be used for more general setting in group decision making
  - Example: determine whether to extend this class 30 minutes.
    - You declare your monetary value for the choice (e.g., for extending, $20, -$10, etc., assuming not change is $0)
    - How can we guarantee that you will declare your true preference?

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Re-distribution of Clarke Tax

- What happens if there is no auctioneer?

Example:

- A group of people is sharing a car.
- They need to decide who is going to use the car in this weekend.
- If everybody declares his/her utility of using the car, and the car is allocated by Vickrey/second-price auction, then we can guarantee the mechanism is dominant-strategy incentive compatible and the allocation is Pareto efficient.
  - e.g., member 1: $100, member 2: $80, member 3: $60, member4: $40, then member 1 uses the car and pays $80.
- However, burning $80 is wasteful!

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Re-distribution Method

Requirement: honesty is a dominant strategy, re-distributed as much as possible, the mechanism does not make any loss.

Idea 1: split the revenue (re-distribute $80/4=$20)

- Member 2 has an incentive to over-bid.

Idea 2: split the revenue except member 2

- Member 2 would be better off by under-bid and becomes the third (and receive re-distribution)

Idea 3: Member 2 receives the value of the third/4, others receive the value of the second/4 --- enough money.

- Almost done, but there is a case that member 2 has an incentive to over-bid to become the winner.

1: $100, 2: $80, 3: $60, 4: $40 ---
member 1 uses the car and pays $80.

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Re-distribution (correct)

- Members 1 and 2 receive the value of the third/4 = $60/4 = $15, the rest receive the value of the second/4 = $80/4 = $20.
- The total amount of re-distribution is $70, thus, $10 must be burned.
- There exists no mechanism that always achieves a Pareto efficient allocation and re-distribute all money

1: $100, 2: $80, 3: $60, 4: $40 ---
member 1 uses the car and pays $80.

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Can We Re-distribute All?

It is possible if we give up a Pareto efficient allocation.

- Choose one member i at random.
- Member i cannot use the car.
- Apply Vickrey/second-price among the rest of members.
- The payment is given to member i.
- No need to burn money.
- If member i's valuation is highest, the allocation is not Pareto efficient.

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Further Readings (books)

- Textbook on Auction:
  - Vijay Krishna, Auction Theory, Academic Press, 2002.
- Textbook on Combinatorial Auctions
  - Combinatorial Auctions, Peter Crampton, Yoav Shoham, Richard Steinberg, eds., MIT Press, 2006.
- Mid-level textbooks on economics in general:
  - Andreu Mas-Colell, Michael D. Whinston and Jerry R. Green, Microeconomic Theory, Oxford University Press, 1995.