False-name-proof Multi-unit Auction Protocol Utilizing Greedy Allocation Based on Approximate Evaluation Values

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ABSTRACT

This paper presents the new false-name-proof multi-unit auction protocol called Greedy ALlocation (GAL) protocol. Although Internet auctions have been growing very rapidly, the possibility of a new type of cheating called false-name bids has been identified.False-name bids are bids made under fictitious names, e.g., multiple e-mail addresses. A protocol called the Iterative Reducing (IR) protocol has been developed for multi-unit auctions and proven to be false-name-proof, i.e., using false-name bids is useless. For Internet auction protocols, being false-name-proof is important since identifying each participant is virtually impossible.

One shortcoming of the IR protocol is that it requires the auctioneer to carefully pre-determine a reservation price for one unit. Our newly developed GAL protocol is easier to use than the IR, since the auctioneer does not need to set a reservation price. The results of evaluation show that the GAL protocol obtains a social surplus that is very close to Pareto efficient. Furthermore, the obtained social surplus and seller's revenue are much greater than with the IR protocol even if the reservation price is set optimally.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multiagent systems*; K.4.4 [**Computers and Society**]: Electronic Commerce

General Terms

Economics, Theory

Keywords

multi-unit auction, mechanism design, false-name-proof

1. INTRODUCTION

Internet auctions have become an especially popular part of Electronic Commerce (EC) and a promising field for applying AI technologies. The Internet provides an excellent infrastructure for executing much cheaper auctions with many more sellers and buyers from all over the world. However, the authors pointed out the possibility of a new type of cheating called *false-name bids*, i.e., an agent may try to profit from submitting false bids made under fictitious names, e.g., multiple e-mail addresses [7, 12, 15]. Such a dishonest action is very difficult to detect since identifying each participant on the Internet is virtually impossible. Compared with collusion [6, 11], a false-name bid is easier to execute since it can be done by someone acting alone, while a bidder has to seek out and persuade other bidders to join in collusion. We can consider false-name bids as a very restricted subset of general collusion.

Auctions can be classified into three types according to the numbers of items and units auctioned: (i) single item, single unit, (ii) single item, multiple units, and (iii) multiple items. The authors have been conducted a series of works on false-name bids. The results can be summarized as follows.

- For multi-unit auctions, where the demand of a participant can be multiple units, or for combinatorial auctions of multiple items, the generalized Vickrey auction protocol (GVA) [11] is not false-name-proof.
- There exists no false-name-proof auction protocol that simultaneously satisfies Pareto efficiency and individual rationality in the above situations.
- We developed a false-name-proof combinatorial auction protocol called the LDS protocol [13], and a false-name-proof multi-unit auction protocol called the IR protocol [14].

In this paper, we concentrate on private value auctions [5]. In private value auctions, each agent knows its own evaluation values of goods with certainty, which are independent of the other agents' evaluation values. We define an agent's utility as the difference between this private value of the allocated goods and its payment. Such utility is called *quasi-linear* utility [5]. These assumptions are commonly used for making theoretical analyses tractable.

In a traditional definition [5], an auction protocol is (dominant-strategy) incentive compatible (or *strategy-proof*), if bidding of the true private values of goods is the dominant strategy for each agent, i.e., is the optimal strategy regardless of the actions of other agents. The revelation principle states that in the design of an auction protocol we can restrict our attention to incentive compatible protocols without loss of generality [5]. In other words, if a certain property (e.g., Pareto efficiency) can be achieved by using some auction protocol in a dominant strategy equilibrium, i.e., a combination of the dominant strategies of agents, the property can also be achieved by using an incentive compatible auction protocol.

In this paper, we extend the traditional definition of incentive-compatibility so that it can address false-name bid manipulations, i.e., we define that an auction protocol is (dominant-strategy) incentive compatible, if bidding of the true private values of goods through the true identifier is the dominant strategy for each agent. To distinguish between the traditional and extended definitions of incentive-compatibility, we refer to the traditional definition as strategy-proof, and the extended definition as *false-name-proof*. In [12, 15], one of the authors showed that the revelation principle still holds for false-name-proof protocols, i.e., we can restrict our attention to false-name-proof protocols without loss of generality.

We say that an auction protocol is Pareto efficient when the sum of all participants' utilities (including that of the auctioneer), i.e., the social surplus, is maximized in the dominant strategy equilibrium. An auction protocol is individually rational if no participant suffers any loss in the dominant strategy equilibrium, i.e., the payment never exceeds the evaluation value of the goods obtained. In a private value auction, individual rationality is indispensable; no agent wants to participate in an auction where it might be charged more money than it is willing to pay. Therefore, in this paper, we restrict our attention to individually rational protocols.

In this paper, we concentrate on multi-unit auctions, in which multiple units of an identical item are sold. Multi-unit auctions have practical importance and are widely executed already in current Internet auction sites such as eBay and Yahoo!. In Internet auctions at present, a participant is assumed to want only one unit of an item. By allowing a participant to bid on multiple units, e.g., he/she needs two units of an item at the same time, as in combinatorial auctions [8, 4], we can increase both the utility of the participants and the revenue of the seller.

The GVA protocol [11] is one instance of the well-known Clarke mechanism [5]. This protocol is strategy-proof, Pareto efficient, and individual rational in multi-unit auctions where there are no false-name bids. However, the GVA is no longer false-name-proof for a multi-unit auctions if the marginal utility of a unit may increase [7, 15]. The marginal utility of an item means the increase in the agent's utility as a result of obtaining one additional unit. When the number of units becomes very large, the marginal utility of a unit tends to decrease. For example, if we already have one million units of an item, the utility of having additional one unit would be close to zero. On the other hand, if the number of units are relatively small, which is common in many auction settings, we cannot assume that the marginal utility of each agent always decreases. A typical example where the marginal utility increases is the all-or-nothing case, where an agent needs a certain number of units, otherwise the good is useless (e.g., airplane tickets for a family trip). Also, in application level network bandwidth auctions, each agent tries to obtain bandwidth for its application. As discussed in [9], we cannot assume the marginal utility always decreases. For example, to use a video-conference application, the required bandwidth must be larger than a certain threshold, otherwise, the obtained quality would be too low and the agent would prefer not to have a video-conference at all.

In [14], a false-name-proof multi-unit auction protocol called the Iterative Reducing (IR) protocol is presented. Although this protocol does not satisfy Pareto efficiency, it can achieve a relatively good social surplus. However, in this protocol, the auctioneer must determine a reservation price, i.e., the minimal price for selling one unit of an item. The obtained social surplus and the revenue of the seller critically depends on the reservation price, thus the auctioneer must carefully determine the reservation price to obtain a high social surplus or revenue. Setting an appropriate reservation price is difficult task since it must be determined beforehand, i.e., the price must be independent from actual bids of participants.

In this paper, we develop a new false-name-proof multi-unit auction called Greedy ALlocation (GAL) protocol. This protocol can handle arbitrary evaluation values of agents, including the case where the marginal utility of an agent increases. In the GAL protocol, the auctioneer does not need to set a reservation price nor any other parameters. The characteristics of this protocol are that the auctioneer first approximates the evaluation value of an agent as a step function with multiple steps, in which the average marginal utility decreases. Then, the protocol determines a tentative assignment by allocating units using a greedy method. To prevent a demand reduction lie [1], we are going to allocate a smaller number of units if the utility of agent increases.

We compare the obtained the social surplus and the seller's revenue obtained under the GAL and IR protocols. The evaluation results show that the GAL protocol obtains a social surplus that is very close to Pareto efficient. Furthermore, both the social surplus and seller's revenue are much better than under the IR protocol, even when an optimal reservation price has been set.

2. THE GREEDY ALLOCATION (GAL) PROTOCOL

2.1 Single-Step Case

Let us assume that there are n agents and m units of an identical item. Firstly, let us consider the special case in which for each agent, its evaluation value is all-or-nothing, i.e., the agent requires a certain number of units, otherwise, its evaluation value is 0. Please note that since the marginal utility of a unit increases in this case, the GVA protocol is no longer false-name-proof.

In this case, we can use a simple extension of the $M{+}1$-st auction protocol [5]. As is shown in Figure 1 (a), such an evaluation value can be represented as a step function with

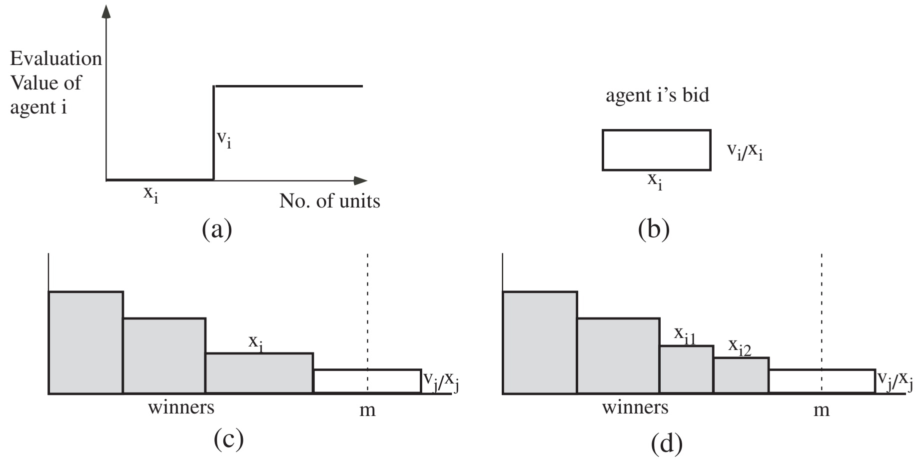

Figure 1: GAL Protocol for Single-Step Case

a single step. Let us represent the number of units required by agent i as x_i, which corresponds to the length to the step from the origin, and the evaluation value of x_i units as v_i, which corresponds to the height of the step.

Now, we are going to represent the agent i's bid as a rectangle of width x_i and height v_i/x_i, i.e., the area of the rectangle is v_i (Figure 1 (b)). Then, we are going to sort bids of all agents by its height, i.e., the average evaluation value of one unit (Figure 1 (c)). We assume that ties are broken through random selection. The winners are agents who submit bids that are placed to the left of m (shaded bids in the figure). Assume that the first rejected bid is agent j's bid, with a height of v_j/x_j. A winner then pays v_j/x_j for each unit (Figure 1 (c)).

It is clear that this protocol is strategy-proof assuming an agent cannot submit false-name bids. For a winner, the price of one unit is determined independently from its declared evaluation value, thus under and over declaring its evaluation value is useless. Also, since we assume the evaluation value of an agent is all-or-nothing, under and over declaring its required units is also useless. Furthermore, for a looser, over-declaring its evaluation value to become a winner is useless, since the price for each unit becomes larger than (or equal to) its average value for one unit.

It is also clear that this protocol is false-name-proof. Assume an agent i uses two identifiers i_1 and i_2, and obtains x_{i_1} units under identifier i_1 and x_{i_2} units under identifier i_2. In this case, as long as $x_i = x_{i_1} + x_{i_2}$, the price per unit is the same as in the case where agent i uses a single identifier (Figure 1 (d)).

A bidder with an all-or-nothing evaluation value is equivalent to a *single-minded* bidder in a combinatorial auction [4]. Also, the GAL protocol for this special case can be considered as one instance of a class of greedy protocols described in [4]. In this paper, we are going to extend the protocol so that it can handle the general case where the evaluation value of an agent is an arbitrary step function with multiple steps (i.e., not single-minded).

2.2 The Multiple-Step Case with Decreasing Average Marginal Utility

Now, let us consider the case where the evaluation value of an agent may consist of multiple steps, but the average marginal utility decreases (Figure 2 (a)). The average marginal utility is the gradient of each step, i.e., $v_{i,t}/x_{i,t}$, where $x_{i,t}$ is the t-th required number of units of agent i, and $v_{i,t}$ is the increase of agent i's utility for obtaining additional $x_{i,t}$ units. Please note that the assumption that *average* marginal utility decreases is more general than the assumption that marginal utility decreases for all units, i.e., if the marginal utility decreases for all units, then it is obvious that the *average* marginal utility decreases, but not vice versa. For example, the assumption that average marginal utility decreases includes the single-step (i.e., all-or-nothing) case, which is a typical example where the marginal utility increases.

In this case, we can represent agent i's bids as multiple rectangles, each of width $x_{i,t}$ and height $v_{i,t}/x_{i,t}$ (Figure 2 (b)). By the assumption of decreasing average marginal utility, the heights of these rectangles decrease, i.e., for all t, $v_{i,t}/x_{i,t} \geq v_{i,t+1}/x_{i,t+1}$ holds. Basically, we can apply the GAL protocol for the single step case, i.e., we sort all of the bids and determine the winners, and the price per unit is determined by the height of the first rejected bid. However, we need to consider the possibility of a demand reduction lie [1]. Let us consider the following example.

EXAMPLE 1. *There are three agents* $1, 2,$ *and* 3. *There are two units of an item. The evaluation value of agent* 1 *for the first unit is* 100, *and the marginal utility for the second unit, i.e., obtaining one additional unit when it already has one unit is* 71. *Agent* 2 *requires only one unit and its evaluation value is* 70. *Agent* 3 *requires only one unit and its evaluation value is* 20. *In this case, by applying the simple extension of the above protocol, agent* 1 *obtains two units at a price per unit of* 70. *Therefore, the utility of agent* 1 *is* $171 - 70 \times 2 = 31$. *On the other hand, if agent* 1 *declares the marginal utility of a second unit to be* 0 *(i.e., agent* 1 *lies that it only needs one unit), agent* 1 *and agent* 2 *obtain one unit each and the price per unit becomes* 20. *The utility of agent* 1 *is now* $100 - 20 = 80$.

To prevent a demand reduction lie, we add the following procedure to the original protocol.

- For each winner i, which can obtain k units, we cal-

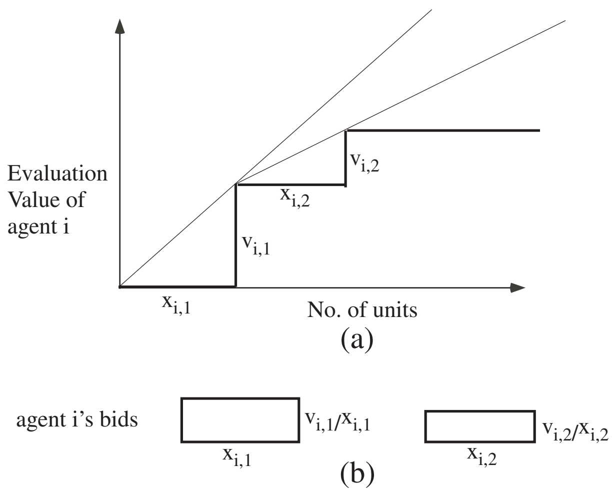

Figure 2: GAL Protocol for the Multiple-Step Case with Decreasing Average Marginal Utility

culate the payment and the utility of agent i when it uses a demand reduction lie and obtains $k-1$ units, $k-2$ units, and so on, and apply the result for agent i that maximizes its utility.

In the above example, agent 1's utility is maximized when it obtains one unit by paying 20, so we allocate one unit to agent 1.

Please note that if agent i prefers to obtain fewer units at a lower price, we are not going to sell the remaining units (that agent i refuses to buy) to other agents. In this example, we are not going to sell the remaining unit to agent 2. This is required to guarantee that the protocol is strategy-proof. More specifically, if we allocate the remaining units to other agents, an agent might have an incentive for over-declaring its evaluation value so that the price of other agents increases. As a result, the other agents will prefer to buy a smaller numbers of units, so the first agent can obtain a larger number of units.

By this extension, this protocol is guaranteed to be strategy-proof. This is because for each agent, under-declaring its evaluation value is useless since if a demand reduction lie is effective, the protocol applies the result when the agent truthfully declares its evaluation value. Furthermore, this protocol is still false-name-proof since the same argument as the single step case is still valid.

2.3 General Case

The GAL protocol can be extended to the general case where the evaluation value of an agent is an arbitrary step function with multiple steps (Figure 3 (a)). In this case, we are going to approximate the evaluation value of agent i using a step function with multiple steps, which satisfies the condition of decreasing average marginal utility (Figure 3 (a)) and represent agent i's bids as multiple rectangles (Figure 3 (b)). Please note in this general case, we can represent arbitrary evaluation values including the case where the marginal utility of an agent increases. The only restriction we need is that the evaluation value is non-decreasing. This assumption is quite natural and can be automatically satisfied if we assume free-disposal.

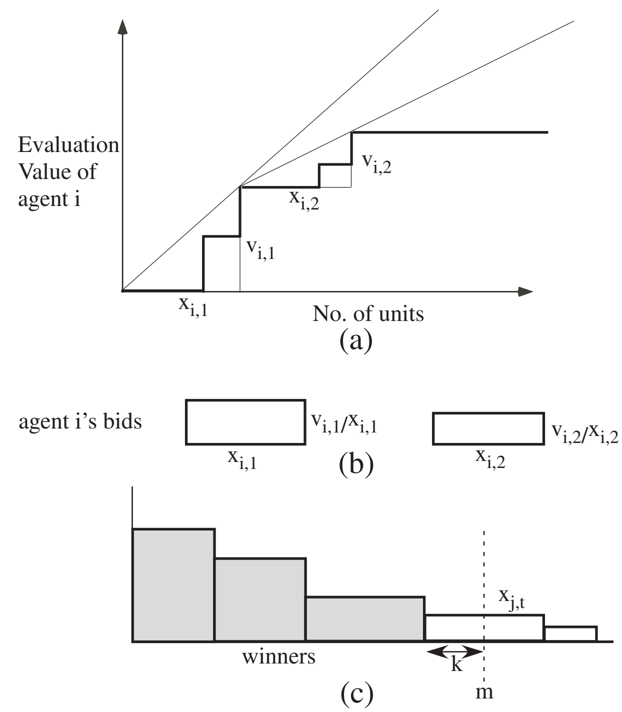

Figure 3: GAL Protocol for the General Case

The only change we need to make concerns the treatment on first rejected bid. In Figure 3 (c), agent i's second bid with width $x_{i,2}$ and height $v_{i,2}/x_{i,2}$, is the first rejected bid. In this case, agent i might be willing to buy the remaining l or fewer units, where $l < x_{i,2}$, since i's real evaluation value is not all-or-nothing for additional $x_{i,2}$ units. Therefore, we consider the possibility that agent i obtains additional l or fewer units and choose the number of units so that agent i's utility is maximized.

Please note that this approximate evaluation function of agent i is used only for determining the tentative assignments and the prices for agents except i. The actual number of allocated units for agent i is calculated based on the real evaluation value.

We give a precise description of the protocol in Appendix A and the proof that the GAL protocol is false-name proof in Appendix B.

3. THE ITERATIVE REDUCING (IR) PROTOCOL

We briefly describe the IR protocol [14], which is the only existing non-trivial protocol that can handle multi-unit auctions. In the IR protocol, the auctioneer must pre-define a reservation price r for each unit. The IR protocol determines the allocations of units sequentially from larger sets. More specifically, the protocol first check whether there exists an agent whose evaluation value for m units is larger than the reservation price $r \times m$. If not, the protocol tries to sell $m-1$ units, and so on. When some bundles of k units are allocated, and there exist enough remaining units, the protocol continues to sell a smaller set of units. Please consult [14] for the details of the IR protocol.

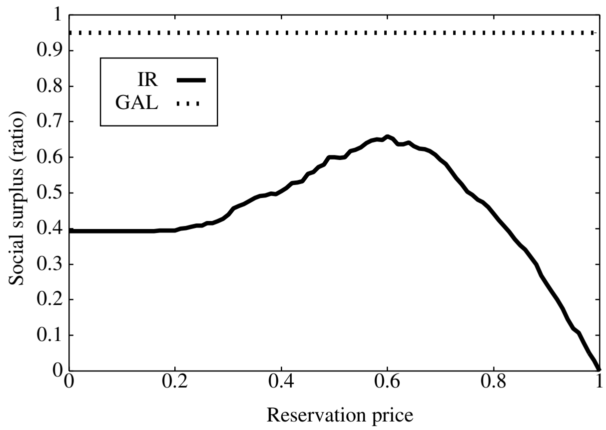

Figure 4: Comparison of Social Surplus (Single Step)

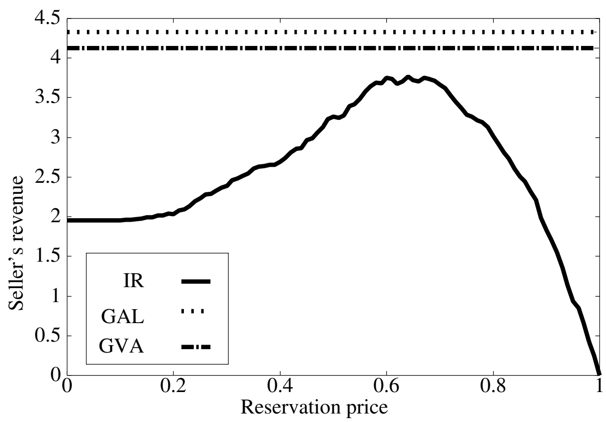

Figure 5: Comparison of Seller's Revenue (Single Step)

One shortcoming of the IR protocol is that the auctioneer must determine the reservation price r. The obtained social surplus and the revenue of the seller critically depend on the reservation price. Setting an appropriate reservation price is difficult task since it must be determined beforehand, i.e., the price must be independent from actual bids of participants.

4. EVALUATIONS

In this section, we report on our use of simmulation to compare the obtained social surplus and seller's revenue of the GAL protocol with that of the IR protocol by simulation.

Firstly, we use the identical setting used in [14]. In this setting, the evaluation value of each agent is represented as a step function with a single step, i.e., all-or-nothing.

The method of generating the evaluation value of agent i is described as follows.

- Determine the number of required units x_i that agent i wants to have by using a binomial distribution $B(m,p)$, i.e., the probability that the number is x_i is given by $p^{x_i}(1-p)^{m-x_i}m!/(x_i!(m-x_i)!)$.
- Randomly choose v_i, i.e., i's evaluation value for x_i units, from within the range of $[0, x_i]$. We assume that the evaluation value of an agent is all-or-nothing, i.e., the evaluation value for obtaining more units than x_i is still v_i, i.e., having additional units is useless.

We generated 100 problem instances by setting the number of agents $n = 10$, the number of units $m = 10$, and $p = 0.2$. Figure 4 shows the average ratio of the obtained social surplus to the Pareto efficient social surplus by varying the reservation price. We can see that this ratio of the social surplus for the GAL protocol reaches around 95%, which is much better than the result for the IR protocol (less than 70%) even when the reservation price is chosen optimally.

Figure 5 shows the average of the obtained seller's revenue by varying the reservation price. We give the results of the IR and GVA protocols for comparison, where we assume there exists no false-name bids. We can see that the average of the seller's revenue under the GAL protocol is around 4.33, which is better than the result under the GVA (around 4.13) and much better than that under the IR protocol (less than 3.78) even when the reservation price is chosen optimally.

Next, we use another setting where an evaluation value of an agent can have multiple steps. As discussed in [2], such an assumption is quite natural if we consider the case where a necessary bandwidth for some application (e.g., a video-streaming service) is obtained by auctions.

The method of generating the evaluation value is as follows. We generate each step of an agent according to the same method as was used in the previous setting. When generating the t-th step, where the number of required units is $x_{i,t}$, we randomly choose $v_{i,t}$ from within the range of $\gamma^{t-1}[0, x_{i,t}]$, where γ is a parameter to control the degree that average marginal utility decreases. Note that even when γ becomes small, the average marginal utility can still increase.

We generated 100 problem instances by setting the number of agents $n = 100$, the number of units $m = 100$, $p = 0.05$, and $\gamma = 0.8$. We set the maximal number of steps in an evaluation value to 3. Figure 6 shows the average ratio of the obtained social surplus to the Pareto efficient social surplus by varying the reservation price. We can see that this ratio of social surplus for the GAL protocol reaches 97%, which is better than the results for the IR protocol even when the reservation price is set optimally. Figure 7 shows the average of the obtained seller revenue by varying the reservation price. We can see that the average of the seller's revenue under the GAL protocol is around 77.1, which is very close to the result under the GVA (around 79.4) and much better than that of the IR protocol (less than 70) even when the reservation price is chosen optimally.

5. DISCUSSIONS

We have shown that the GAL protocol outperforms the IR protocol and performs as well as the GVA regarding the obtained social surplus and seller's revenue. Please note that the evaluation results of the GVA given in the previous section were obtained under the assumption that there exists no false-name bid. With the possibility of the false-name bids, truth-telling is no longer a dominant strategy in the GVA and we cannot predict the outcome of the auction.

If there is no possibility of false-name bids, we are able to

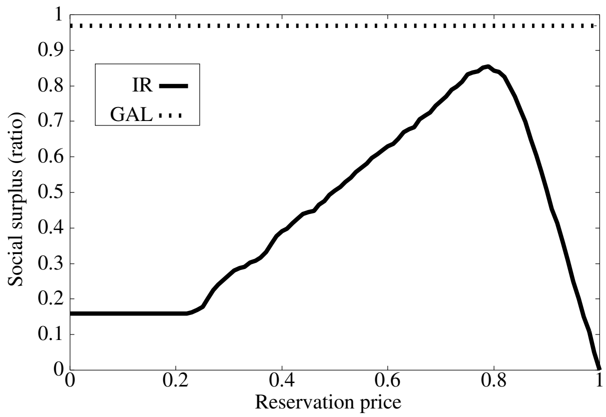

Figure 6: Comparison of Social Surplus (Multiple Steps)

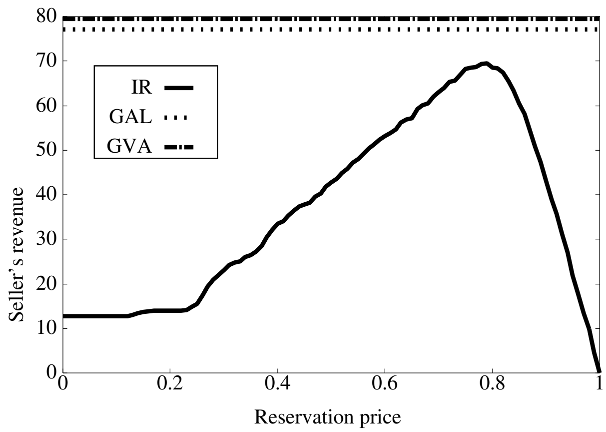

Figure 7: Comparison of Seller's Revenue (Multiple Steps)

use the GVA for multi-unit auctions. In that case, the obtained social surplus is optimal, i.e., Pareto efficient. However, the computational cost for applying the GVA is not very small. Although we can use a dynamic programming technique for finding the optimal allocation, where the computational cost is $O(n \cdot m^2)$ [10], we need to repeatedly solve an optimization problem to determine the payment of each winner.

On the other hand, the computational cost of the GAL protocol is much smaller since the GAL protocol uses a greedy algorithm to find a semi-optimal allocation and the price per unit for each agent can be determined without solving a combinatorial optimization problem. This will be an advantage of the GAL protocol when m becomes large and we need to apply the protocol repeatedly to re-allocate resources, which is the case in some network bandwidth auctions [3].

6. CONCLUSIONS

In this paper, we have presented a new false-name-proof multi-unit auction protocol (GAL protocol). In the GAL protocol, the auctioneer does not need to set the reservation price nor any other parameters. The characteristics of this protocol are that the auctioneer uses an approximate evaluation values and a greedy algorithm to determine a tentative assignment and prices per unit. We showed that the GAL protocol outperforms the IR protocol and performs as well as the GVA regarding the obtained the social surplus and seller's revenue. Our future works include the application of the GAL protocol to network bandwidth auctions.

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APPENDIX

A. DETAILS OF GAL PROTOCOL

Let us assume there are n agents and m units of an identical item. Each agent declares its (not necessarily true) evaluation value for j units as $v(i,j)$, where its true evaluation value for j units is $u(i,j)$.

The GAL protocol is executed in the following three phases.

Approximation of evaluation values:

For each agent i, set $x_{i,1}$ to $\arg\max_j v(i,j)/j$, $v_{i,1}$ to $v(i, x_{i,1})$, and $b_{i,1}$ to $v_{i,1}/x_{i,1}$. This procedure corresponds to find the step where the average marginal utility, i.e., the gradient is maximized in Figure 3 (a).

Then, let us define the marginal utility after $x_{i,1}$ units are allocated as $v_1(i,j)$. More specifically, $v_1(i,j)$ is defined as $v(i, x_{i,1}+j) - v(i, x_{i,1})$. By using $v_1(i,j)$, we set $x_{i,2} = \arg\max_j v_1(i,j)/j$, $v_{i,2}$ to $v_1(i, x_{i,2})$, and $b_{i,2}$ to $v_{i,2}/x_{i,2}$, and so on.

We represent the t-th bid of agent i as a rectangle denoted by $(b_{i,t}, x_{i,t})$, where $b_{i,t}$ is the height and $x_{i,t}$ is the width.

Determining tentative allocation:

Let us denote $\mathbf{B}$ as a list of all rectangles, in which rectangles are sorted by their heights in a decreasing order (ties are broken randomly). Let us represent the t-th element of $\mathbf{B}$ as $(b_{(t)}, x_{(t)})$. Let us define $S(t) = \sum_{1\le u\le t} x_{(u)}$. $S(t)$ represents the sum of the width from the first to the t-th bids.

Next we define $maxu(i)$, which denotes the tentative (actually the maximal) number of units allocated to agent i. Let us choose k where $S(k) \le m$ and $S(k+1) > m$ hold, i.e., the first rejected bid is the $k+1$-th bid.

case 1: if $(b_{(k+1)}, x_{(k+1)})$ is not agent i's bid,
$maxu(i) = \sum_{1\le t\le k} x_{(t)}$, where $(b_{(t)}, x_{(t)})$ is i's bid.

case 2: if $(b_{(k+1)}, x_{(k+1)})$ is i's bid,
$maxu(i) = maxu'(i) + m - S(k)$, where $maxu'(i) = \sum_{1\le t\le k} x_{(t)}$ and $(b_{(t)}, x_{(t)})$ is i's bid,

Determining the actual number of units:

Let us denote $\mathbf{B}^{\sim \mathbf{i}}$ as a sorted list that excludes the bids of agent i. We represent the t-th element of $\mathbf{B}^{\sim \mathbf{i}}$ as $(b^{\sim i}_{(t)}, x^{\sim i}_{(t)})$. Also, let us define $S^{\sim i}(t) = \sum_{1\le u\le t} x^{\sim i}_{(u)}$.

Also, for each agent i, for each j where $1 \le j \le m$, we define $k^{\sim i}(j)$ as t where $S^{\sim i}(t) \le m-j$ and $S^{\sim i}(t+1) > m-j$ hold. The intuitive meaning of $k^{\sim i}(j)$ is that, assuming agent i obtains j units, then $k^{\sim i}(j)$-th bid in $\mathbf{B}^{\sim \mathbf{i}}$ can be placed to the left of m, while $k^{\sim i}(j)+1$-th bid will be the first rejected bid.

For each agent i, i's payment per unit $p_{i,j}$ where j units are allocated to agent i is decided as follows.

$$p_{i,j} = b^{\sim i}_{(k^{\sim i}(j)+1)}$$

The allocated number j^*_i is determined as follows.

$$j^*_i = \arg\max_j v(i,j) - p_{i,j} \times j, \text{where } 0 \le j \le maxu(i).$$

As a result, agent i obtains j^*_i units and pays $j^*_i \times p_{i,j^*_i}$.

B. PROOF: GAL IS FALSE-NAME-PROOF

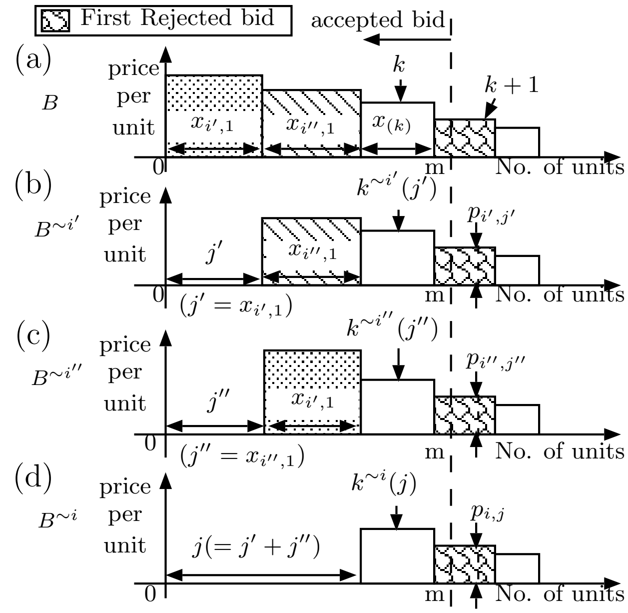

Figure 8: Comparison of the Position of Price Decision Bids

We show that the following theorem holds.

THEOREM 1. *The GAL protocol is false-name-proof.*

We first show the following lemma holds.

LEMMA 1. *If an agent uses multiple identifiers, it can obtain the same number of units with a smaller (or equal) payment by using a single identifier, i.e., an agent cannot increase its utility by submitting false-name bids.*

To show Lemma 1 holds, we first prove the following lemma.

LEMMA 2. *If an agent i uses two identifiers i' and i'', and obtains j' and j'' units under the identifiers i' and i'', respectively, then agent i can obtain $j = j' + j''$ units at a payment that is less than (or equal to) the sum of the payments for j' and j'' by using a single identifier.*

The proof is as follows. Without loss of generality, we can assume that agent i' has only one bid that can be represented as a rectangle with width j', and that agent i'' has only one bid that can be represented as a rectangle with width j''. This is because the price per unit for an agent is invariant to the declared evaluation value of the agent. Also, if the width of the rectangle becomes wider or other bids are added, the price for another identifier never decreases.

As described in Appendix A, $\mathbf{B}$ is a list of all rectangles (sorted by their heights) (Figure 8 (a)), and $(b_{(k)}, x_{(k)})$is the last accepted bid and $(b_{(k+1)}, x_{(k+1)})$ is the first rejected bid, i.e., $S(k) \le m$ and $S(k+1) > m$ hold.

The price per unit if agent i' obtains j' units is given by $p_{i',j'} = b^{i'}_{(k^{\sim i'}(j')+1)}$, where $k^{\sim i'}(j')$ is chosen so that $S^{\sim i'}(k^{\sim i'}(j')) \le m - j'$ and $S^{\sim i'}(k^{\sim i'}(j')+1) > m - j'$ holds, i.e., $k^{\sim i'}(j')+1$-th bid is the first rejected bid in $\mathbf{B}^{\sim \mathbf{i'}}$ for allocating $m - j'$ units.

Since $\mathbf{B}^{\sim \mathbf{i}'}$ is identical to $\mathbf{B}$ except the rectangle of agent i' (who has only one rectangle), $S(k) = S^{\sim i'}(k-1) + j'$ and $S(k+1) = S^{\sim i'}(k) + j'$ hold. From $S(k) \leq m$ and $S(k+1) > m$, we obtain $S^{\sim i'}(k-1) \leq m - j'$ and $S^{\sim i'}(k) > m - j'$, which means $k^{\sim i'}(j') = k - 1$(Figure 8 (b)).

In summary, $p_{i',j'}$ is equal to the height of k-th bid in $\mathbf{B}^{\sim \mathbf{i}'}$, which is identical to $k+1$-th bid in $\mathbf{B}$, i.e., $p_{i',j'} = b_{(k+1)}$ (Figure 8 (c)).

Similarly, we can show that the price per unit if agent i'' obtains j'' units is given by $p_{i'',j''} = b^{i''}_{(k^{\sim i''}(j'')+1)} = b_{(k+1)}$. Therefore, the total payment becomes $b_{(k+1)} \times (j' + j'')$.

Then, let us assume agent i uses a single identifier and submit a single bid that can be represented as a rectangle with width $j = j' + j''$.

The price per unit if agent i obtains j units is given by $p_{i,j} = b^{i}_{(k^{\sim i}(j)+1)}$, where $k^{\sim i}(j)$ is chosen so that $S^{\sim i}(k^{\sim i}(j)) \leq m - j$ and $S^{\sim i}(k^{\sim i}(j)+1) > m - j$ hold, i.e., $k^{\sim i}(j)+1$-th bid is the first rejected bid in $\mathbf{B}^{\sim \mathbf{i}}$ for allocating $m - j$ units(Figure 8 (d)).

Since $\mathbf{B}^{\sim \mathbf{i}}$ is identical to $\mathbf{B}$ except the rectangles of agent i' and agent i'', $S(k) = S^{\sim i}(k-2) + j$ and $S(k+1) = S^{\sim i}(k-1) + j$ hold. From $S(k) \leq m$ and $S(k+1) > m$, we obtain $S^{\sim i}(k-2) \leq m - j$ and $S^{\sim i}(k-1) > m - j'$, which means that $k^{\sim i}(j) = k - 2$.

In summary, $p_{i,j}$ is equal to the height of the $k-1$-th bid in $\mathbf{B}^{\sim \mathbf{i}}$, which is identical to that of the $k+1$-th bid in $\mathbf{B}$, i.e., $p_{i,j} = b_{(k+1)}$. Therefore, the payment becomes $b_{(k+1)} \times j$, which is equal to that in the case where agent i uses false-names i' and i''. □

Using a similar method of the proof of Lemma 2, we can show that Lemma 1 holds.

Now, we show that the following lemma holds.

LEMMA 3. *If an agent uses a single identifier, truth-telling is the dominant strategy.*

From the definition of the GAL protocol, it is clear that for an agent, under-declaring its evaluation value is useless, since the GAL protocol allocates a smaller number of units if doing so is profitable for the agent. To prove Lemma 3, we show that the following lemma holds.

LEMMA 4. *An agent cannot increase its utility by over-declaring its evaluation value.*

The proof is as follows.

case 1: Firstly, we consider the case where the first rejected bid $(b_{(k+1)}, x_{(k+1)})$ is not from agent i'. For agent i, we assume that bids $(b_{i,1}, x_{i,1}), \ldots, (b_{i,i_k}, x_{i,i_k})$ are accepted and $(b_{i,i_k+1}, x_{i,i_k+1})$ is rejected.

If agent i obtains $j = maxu(i)$ units, the price per unit ,$p_{i,j}$, is given by $p_{i,j} = b^{\sim i}_{(k^{\sim i}(j)+1)} = b_{(k+1)}$, by using a similar argument to the proof of Lemma 2.

Now, let us assume that agent i over-declares its evaluation value and obtain $j+a$ units in total. The true evaluation value for obtaining j units and $j+a$ units are represented as $u(i,j), u(i,j+a)$, respectively.

According to the method for determining the height and width of the rectangles, the following formula holds (otherwise, $b(i, i_k+1)$ must be set to a larger value).

$$u(i, j+a) - u(i,j) \leq b(i, i_k+1) \times a \tag{1}$$

If agent i obtains $j+a$ units, the price per unit ,p', is greater than or equal to $p_{i,j} = b_{(k+1)}$. Also, since $b(i, i_k+1)$ is rejected but is not the first rejected bid, $b(i, i_k+1) \leq b_{(k+1)}$ holds. In summary, the following formula holds.

$$b(i, i_k+1) \leq p_{i,j} = b_{(k+1)} \leq p' \tag{2}$$

The utility for obtaining $j+a$ units by paying p' for each unit is $u(i, j+a) - p' \times (j+a)$. From formulae (1) and (2), we can derive the following condition.

$$\begin{aligned} & u(i,j+a) - p' \times (j+a) \\ \leq \;& u(i,j+a) - p_{i,j} \times (j+a) \\ \leq \;& u(i,j+a) - p_{i,j} \times a - p_{i,j} \times j \\ \leq \;& u(i,j+a) - b(i,i_k+1) \times a - p_{i,j} \times j \\ \leq \;& u(i,j) - p_{i,j} \times j \end{aligned}$$

Since $u(i,j) - p_{i,j} \times j$ is the utility for obtaining j units by paying $p_{i,j}$ for each unit, the above condition shows that the utility for obtaining j units is larger than or equal to the utility for obtaining $j+a$ units.

If agent i truthfully declares its evaluation value, it can obtain j units. Thus, the agent cannot increase its utility by over-declaring its evaluation value.

case 2: Next, we consider the case where the first rejected bid $(b_{(k+1)}, x_{(k+1)})$ is agent i's bid. For agent i, we assume that bids $(b_{i,1}, x_{i,1}), \ldots, (b_{i,i_k}, x_{i,i_k})$ are accepted and that $(b_{i,i_k+1}, x_{i,i_k+1}) = (b_{(k+1)}, x_{(k+1)})$ is the first rejected bid.

If agent i obtains $j = maxu'(i)$ units, where $maxu'(i) = \sum_{1 \leq a \leq i_k}(x_{(i,a)})$, the price per unit ,$p_{i,j}$, is given by $p_{i,j} = b^{\sim i}_{(k^{\sim i}(j)+1)}$.

By a similar argument to case 1, $p_{i,j} = b^{\sim i}_{(k^{\sim i}(j)+1)} \leq b_{(k+2)}$ holds. If $b_{(k+2)}$ is not agent i's bid, $p_{i,j} = b_{(k+2)}$.

Now, let us assume that agent i over-declares its evaluation value and obtain $j'+a$ units in total, where $j' = maxu(i) = maxu'(i) + m - S(k)$. By the method for determining the height and width of the rectangles, the following formula holds (otherwise, $b(i, i_k+1)$ must be set to a larger value).

$$u(i, j'+a) - u(i,j) \leq b(i, i_k+1) \times (j'+a-j) \tag{3}$$

By a similar argument to case 1, the following formula holds.

$$p_{i,j} \leq b_{(k+2)} \leq b_{(k+1)} = b(i, i_k+1) \leq b_{(k)} \leq p' \tag{4}$$

The utility for obtaining $j'+a$ units by paying p' for each unit is $u(i, j'+a) - p' \times (j'+a)$. From formulae (3) and (4), we can derive the following condition.

$$\begin{aligned} & u(i,j'+a) - p' \times (j'+a) \\ \leq \;& u(i,j'+a) - b_{(k+1)} \times (j'+a) \\ \leq \;& u(i,j'+a) - b_{(k+1)} \times (j'+a-j) - b_{(k+1)} \times j \\ \leq \;& u(i,j) - b_{(k+1)} \times j \\ \leq \;& u(i,j) - p_{i,j} \times j \end{aligned}$$

Since $u(i,j) - p_{i,j} \times j$ is the utility for obtaining j units by paying $p_{i,j}$ for each unit, the above formula shows that the utility for obtaining j units is greater than or equal to the utility for obtaining $j'+a$ units.

If agent i truthfully declare its evaluation value, it can obtain at most $maxu(i)$ units. Agent i can choose to obtain j units, where $j = maxu'(i) \leq maxu(i)$ in the GAL protocol. Thus, the agent cannot increase its utility by over-declaring its evaluation value. □