Robust Combinatorial Auction Protocol against False-name Bids

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Abstract

This paper presents a new combinatorial auction protocol that is robust against false-name bids. Internet auctions have become an integral part of Electronic Commerce (EC) and a promising field for applying agent and Artificial Intelligence technologies. Although the Internet provides an excellent infrastructure for combinatorial auctions, we must consider the possibility of a new type of cheating, i.e., an agent tries to profit from submitting several bids under fictitious names (false-name bids). If there exists no false-name bid, the Generalized Vickrey Auction protocol (GVA) satisfies individual rationality, Pareto efficiency, and incentive compatibility. On the other hand, when false-name bids are possible, it is theoretically impossible for a combinatorial auction protocol to simultaneously satisfy these three properties.

Our newly developed Leveled Division Set (LDS) protocol, which is a modification of the GVA, utilizes reservation prices of auctioned goods for making decisions on whether to sell goods in a bundle or separately. The LDS protocol satisfies individual rationality and incentive compatibility even if agents can submit false-name bids, although it is not guaranteed to achieve a Pareto efficient social surplus. Simulation results show that the LDS protocol can achieve a better social surplus than that for a protocol that always sells goods in one bundle.

Key words: Mechanism Design, Auction, Game Theory, Electronic Commerce

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1 Introduction

Internet auctions have become an especially popular part of Electronic Commerce (EC). Various theoretical and practical studies on Internet auctions have already been conducted [6,10,13]. Among these studies, those on combinatorial auctions have lately attracted considerable attention [2,3,8,11]. Although conventional auctions sell a single good at a time, combinatorial auctions sell multiple goods with interdependent values simultaneously and allow the bidders to bid on any combination of goods.

In a combinatorial auction, a bidder can express complementary/substitutional preferences over multiple bids. For example, in the Federal Communications Commission (FCC) spectrum auction [5], a bidder may desire licenses for adjoining regions simultaneously (i.e., these licenses are complementary), while he/she is indifferent to which particular channel he/she receives (channels are substitutional). By taking into account such complementary/substitutional preferences, we can increase the participants' utilities and the revenue of the seller.

Although the Internet provides an excellent infrastructure for executing combinatorial auctions, we must consider the possibility of new types of cheating. For example, an agent may try to profit from submitting false bids under fictitious names. Such a dishonest action is very difficult to detect since identifying each participant on the Internet is virtually impossible. We call a bid made under a fictitious name a *false-name bid*. The problems resulting from collusion have been discussed by many researchers [3,7,10,12]. Compared with collusion, a false-name bid is easier to execute since it can be done alone, while a bidder has to seek out and persuade other bidders to join in collusion. We can consider false-name bids as a very restricted subclass of collusion.

The authors have analyzed the effects of false-name bids on auction protocols [9,14]. The obtained results can be summarized as follows.

- The Generalized Vickrey Auction protocol (GVA) [12], which has been proven to satisfy the three properties described below, i.e., individual rationality, Pareto efficiency, and incentive compatibility, if there exists no false-name bid, fails to satisfy incentive compatibility nor Pareto efficiency when false-name bids are possible.
- There exists no combinatorial auction protocol that simultaneously satisfies incentive compatibility, Pareto efficiency, and individual rationality for all cases if agents can submit false-name bids.
- The revelation principle [4] still holds even if the agents can submit false-name bids.

In this paper, we concentrate on private value auctions [4,7]. In private value auctions, each agent knows its own preference, and its evaluation value of goods is independent of the other agents' evaluation values. We define an agent's utility as the difference between the true private value of the allocated goods and the payment for the allocated goods. Such a utility is called a *quasi-linear* utility [4,7]. These assumptions are commonly used for making theoretical analyses tractable.

In a traditional definition [4], an auction protocol is (dominant strategy) incentive compatible, if bidding the true private values of goods is the dominant strategy for each agent, i.e., the optimal strategy regardless of the actions of other agents. The revelation principle states that in the design of an auction protocol we can restrict our attention only to incentive compatible protocols without the loss of generality [4]. In other words, if a certain property (e.g., Pareto efficiency, individual rationality) can be achieved in a dominant strategy equilibrium using some auction protocol, the property can also be satisfied in a dominant strategy equilibrium using an incentive compatible auction protocol.

In this paper, we extend the traditional definition of incentive-compatibility so that it can address false-name bid manipulations, i.e., we define that an auction protocol is (dominant strategy) incentive compatible, if bidding the true private values of goods by using the true identifier is the dominant strategy for each agent. Also, we say that auction protocols are robust against false-name bids if each agent cannot obtain additional profit by submitting false-name bids. If such robustness is not satisfied, the auction protocol lacks incentive compatibility.

A Pareto efficient allocation means that the goods are allocated to bidders whose evaluation values are the highest, and that the sum of all participants' utilities (including that of the seller), i.e., the social surplus, is maximized. In a more general setting, Pareto efficiency does not necessarily mean maximizing the social surplus. In an auction setting, however, agents can transfer money among themselves, and the utility of each agent is quasi-linear; thus the sum of the utilities is always maximized in a Pareto efficient allocation.

An auction protocol is individually rational if no participant suffers any loss, in other words, the payment never exceeds his/her evaluation value of the obtained goods. In a private value auction, individual rationality is indispensable; no agent wants to participate in an auction where it might be charged more money than it is willing to pay.

Since these three properties cannot be satisfied simultaneously, we must give up Pareto efficiency, and consider an auction protocol that satisfies incentive compatibility and individual rationality, while still able to achieve a relatively good social surplus. This is because individual rationality is indispensable, and we can restrict our attention only to incentive compatible protocols without the loss of generality. Also, an incentive compatible protocol has practical advantages in Internet auctions. As shown in [12], a user must explicitly declare his/her preferences to a computerized agent to make it function effectively. However, if we use a standard auction protocol such as the first-price sealed-bid auction protocol [7], the user has to make sure that his/her preferences are kept secret from other agents. If we use an incentive compatible protocol, the knowledge of other participants' preferences is useless; thus we don't have to be concerned about this privacy issue.

In the rest of the paper, we first describe the GVA and show an example where the GVA is not robust against false-name bids (Section 2). Then, we describe our newly developed protocol, called *Leveled Division Set (LDS) protocol* (Section 3), and prove that the LDS protocol satisfies incentive compatibility (Section 4). Furthermore, we show simulation results that demonstrate that this protocol can achieve a better social surplus than a protocol that always sells goods in one bundle (Section 5). Finally, we discuss the merits/demerits of the LDS protocol (Section 6).

2 Generalized Vickrey Auction Protocol (GVA)

The GVA protocol is one instance of the Clarke mechanism [1,4], and a generalized version of the Vickrey, or the second-price sealed-bid auction protocol [4,12]. In the second-price sealed-bid auction protocol, each agent bids for a single good, and the agent with the highest bid wins, and pays the amount of the second highest bid.

Let us represent a set of agents $N = \{1, 2, \ldots, n\}$, and a set of all auctioned goods $M = (1, 2, \ldots, m)$. If we consider the possibility of not selling a good to any agent, there are $(n+1)^m$ possible allocations. Let us represent one possible allocation as m-element vector $G =< g_1, g_2, \ldots, g_m >$, where each element g_i is an integer $0 \leq g_i \leq n$, which means that good i is allocated to agent g_i. If $g_i = 0$, we assume that good i is not sold to any agent. Let us represent the set of all possible allocations as SG.

The outline of the GVA can be described as follows.

(1) Each agent declares evaluation values for each possible allocation $G \in SG$. Let $v_x(G)$ denote agent x's declared evaluation value for the allocation G. The reported evaluation values may or may not be true.
(2) The GVA chooses the allocation G^* that maximizes the social surplus, i.e., the sum of all the agents' declared evaluation values defined below.

$$G^* = \arg\max_{G \in SG}(\sum_{y \in N} v_y(G))$$

(3) The payment of agent x (represented as p_x) is calculated as follows.

$$p_x = \sum_{y \neq x} v_y(G^*_{\sim x}) - \sum_{y \neq x} v_y(G^*)$$

Here, $G^*_{\sim x}$ is the allocation that maximizes the sum of all agents' evaluation values except agent x defined as follows.

$$G^*_{\sim x} = \arg\max_{G \in SG}(\sum_{y \neq x} v_y(G))$$

In the GVA, we can assume that agent x pays the decreased amount of the other agents' social surplus resulting from its participation.

The GVA has been proven to be incentive compatible if there exists no false-name bid [4,12]. The proof is as follows. Since we assume that the utility of each agent is quasi-linear, agent x's utility after the payment is given by the following formula. Let $u_x(G^*)$ denote agent x's true private value for G^*.

$$u_x(G^*) - p_x = u_x(G^*) + \sum_{y \neq x} v_y(G^*) - \sum_{y \neq x} v_y(G^*_{\sim x}) \quad (1)$$

The third term in formula (1), i.e., $\sum_{y \neq x} v_y(G^*_{\sim x})$, is independent of agent x's declaration (assuming that there exists no false-name bid); thus agent x can maximize its utility when the sum of the first and second items in formula (1), i.e., $u_x(G^*) + \sum_{y \neq x} v_y(G^*)$, is maximized.

On the other hand, the optimal allocation G^* is chosen so that the sum of the agents' declared evaluation values are maximized, i.e.,

$$G^* = \arg\max_{G}(v_x(G) + \sum_{y \neq x} v_y(G)). \quad (2)$$

Therefore, agent x can maximize its utility by submitting its true private value, i.e., by setting $v_x(G) = u_x(G)$.

In the following, we show an example of how the GVA works.

Example 1 *Let us assume three agents are participating in an auction of two different goods, A and B, and declare the following evaluation values. The evaluation values of an agent are denoted by a triplet: (the value for A alone, the value for B alone, the value for A and B together).*

- *agent 1: (6, 0, 6)*
- *agent 2: (0, 0, 8)*
- *agent 3: (0, 6, 6)*

The evaluation values of agent 2 are all-or-nothing, i.e., having only one good is useless. In this case, A is allocated to agent 1 and B is allocated to agent 3. The payment of agent 1 is calculated as follows. If agent 1 does not participate, both goods are allocated to agent 2, and the social surplus is 8. *When agent* 1 *does participate, agent 3 obtains B and the social surplus except for agent 1 is* 6. *Therefore, agent 1 pays the difference* $8 - 6 = 2$. *The obtained utility of agent 1 is* $6 - 2 = 4$. *The payment and utility of agent 3 are identical to those of agent 1.*

Now, let us show an example where the GVA is not robust against false-name bids.

Example 2 *Let us assume there are only two agents.*

- *agent 1: (6, 6, 12)*
- *agent 2: (0, 0, 8)*

In this case, both goods are allocated to agent 1. Its payment is calculated as 8, since if agent 1 does not participate, agent 2 obtains both goods and the social surplus is 8, and when agent 1 does participate, agent 1 obtains all goods and the social surplus except for agent 1 is 0.

However, agent 1 can use a false-name 3 and create a situation identical to Example 1. In this case, agent 1 can obtain both goods by paying only $2+2 = 4$. *Therefore, its utility becomes* $12 - 4 = 8$, *which means that agent 1 can make a profit by submitting a false-name bid.*

3 Robust Protocol against False-name Bids

3.1 Basic Ideas

There exists a trivial protocol that can satisfy incentive compatibility even if agents can submit false-name bids, i.e., selling all goods in one bundle and using the second-price sealed-bid auction protocol to determine the winner and its payment. We call this simple protocol the *set protocol*. Clearly, this protocol satisfies incentive compatibility, since only one agent can receive goods and submitting additional bids only increases the payment of the winner.

Selling goods in one bundle makes sense if goods are complementary [1] for all agents, i.e., the utility of a set of goods is larger than the sum of the utilities of having each good separately. However, if goods are substitutional for some agents, i.e., the utility of a set of goods is smaller than the sum of the utilities of having each good separately, the set protocol is wasteful; the social surplus and the revenue of the seller can be significantly worse than that for the GVA.

Let us consider a simple case where there are two goods A and B. To increase the social surplus, we must design a protocol where goods can be sold separately in some cases. To guarantee that the protocol is robust against false-name bids, the following condition must be satisfied.

Proposition 1 *If A and B are sold separately to different agents, the sum of the payments must be larger than the highest declared evaluation value for the set of A and B.*

This is a necessary (but not sufficient) condition to guarantee incentive compatibility, i.e., if this condition is not satisfied, there is a chance that a single agent is using two false-names to obtain these goods. However, designing an incentive compatible protocol satisfying this condition is rather difficult. For example, the following simple protocol does not satisfy incentive compatibility.

Vulnerable Protocol: Determine tentative winners and payments using the GVA. If A and B are sold separately to different agents, and the sum of the payments does not satisfy the condition in Proposition 1, then sell the goods in one bundle, otherwise, use the results of the GVA.

Assume the situation in Example 1. Since the result of the GVA does not satisfy the condition in Proposition 1, both goods are sold to agent 2. However, agent 1 can create the following situation using false-names 4 and 5.

- agent 1: (6, 0, 6)
- agent 2: (0, 0, 8)
- agent 3: (0, 6, 6)
- agent 4: (3, 0, 3)
- agent 5: (0, 5.9, 5.9)

In this case, using the GVA, A is allocated to agent 1 and B is allocated to agent 3, and agent 1 pays 3 and agent 3 pays 5.9; thus the condition in Proposition 1 is satisfied. Clearly, agent 1 prefers this result since it can obtain A by paying 3 and its utility becomes $6 - 3 = 3$, while agent 1 cannot obtain

[1] In some microeconomic studies, the definition that good A and good B are complementary states that if the price of good B increases, the demand for good A decreases, and vice versa. This definition is stricter than the definition used in this paper.

any good when it does not submit false-name bids.

As shown in this example, if an agent can submit false-name bids, manipulating the payments of other agents is rather easy. In this case, agent 1 manipulates the result by forcing agent 3 to pay more by submitting a false-name bid. In essence, we must solve the difficult dilemma of satisfying the above condition on payments without using the second highest evaluation values, which are essential to calculating the payments.

Our newly developed protocol solves this dilemma by utilizing *reservation prices* of goods [7]. The seller does not sell a good if the payment for the good is smaller than the reservation price. Let us assume the reservation prices of A and B are r_A and r_B, respectively. In this new protocol, we sell goods separately only when the highest declared evaluation value for the set is smaller than $r_A + r_B$. Also, the payment for each good is always larger than or equal to the reservation price. Clearly, this protocol satisfies the condition of Proposition 1. In the following, we are going to show how this idea can be introduced to the GVA.

3.2 Leveled Division Set Protocol

We first show the sketch of the LDS protocol. In the LDS protocol, the auctioneer determines a leveled division set (Figure 1). This leveled division set describes the possible way for dividing goods among different agents. In level 1, all goods are sold in one bundle. Goods are divided into smaller bundles as the level increases. The auctioneer chooses the level according to declared evaluation values of agents, then uses the GVA within the level to determine the winners and payments.

In the following, we are going to define several terms and notations. To help readability, we use three different types of parentheses, i.e., (),{}, and [], to represent sets.

- a set of agents $N = \{1, 2, \ldots, n\}$
- a set of all auctioned goods $M = (1, 2, \ldots, m)$
- a division, which is a set of bundles, $D = \{S \mid S \subseteq M\}$, where $\forall S, S' \in D$ and $S \neq S'$, $S \cap S' = \emptyset$ holds [2]
- For each good j, the reservation price r_j is defined.
- For a bundle S, we define $R(S)$ as $\sum_{j \in S} r_j$.

A leveled division set is defined as follows.

[2] Note that we don't require $\bigcup_{S \in D} S = M$ holds, i.e., satisfying $\bigcup_{S \in D} S \subseteq M$ is sufficient.

	case 1	case 2	case 3
level 1	[{(A,B)}]	[{(A,B,C)}]	[{(A,B,C,D)}]
level 2	[{(A),(B)}]	[{(A,B)}, {(B,C)}, {(A,C)}]	[{(A,B,C)}, {(B,C,D)}, {(A,D)}]
level 3		[{(A),(B),(C)}]	[{(A),(D),(B,C)}]

Fig. 1. Examples of Leveled Division Sets

- Levels are defined as $1, 2, \ldots, max\_level$.
- For each level i, a division set $SD_i = [D_{i1}, D_{i2}, \ldots]$ is defined.

A leveled division set must satisfy the following three conditions.

- $SD_1 = [\{M\}]$ — the division set of level 1 contains only one division, which consists of a bundle of all goods.
- For each level and its division set, a union of multiple bundles in a division is always included in a division of a smaller level, i.e., $\forall i > 1, \forall D_{ik} \in SD_i, \forall D' \subseteq D_{ik}$, where $\mid D' \mid \geq 2, S_u = \bigcup_{S \in D'} S$, then there exists a level $j < i$, with a division set SD_j, where $D_{jl} \in SD_j$ and $S_u \in D_{jl}$.
- For each level and its division set, each bundle in a division is not included in a division of a different level[3], i.e., $\forall i, \forall D_{ik} \in SD_i, \forall S \in D_{ik}, \forall j \neq i, \forall D_{jl} \in SD_j, S \notin D_{jl}$.

Figure 1 shows examples of leveled division sets. Case 1 shows one instance where there are two goods (A and B), and case 2 and case 3 show instances where there are three and four goods, respectively.

In the following, let us assume agent 0 is a *dummy agent*, whose evaluation value of the good j is equal to the reservation price r_j. For a division $D = \{S_1, S_2, \ldots, S_l\}$ and one possible allocation of goods $G =< g_1, g_2, \ldots, g_m >$, we say G is allowed under D if the following conditions are satisfied.

(1) Multiple goods that belongs to the same bundle in the division must be allocated to the same agent, i.e., $\forall S \in D$, $\forall i, j \in S$, $g_i = g_j$ holds.
(2) If two goods belong to different bundles in the division, they must be allocated to different agents, except for the case where they are allocated to the dummy agent 0, i.e., $\forall S, S' \in D$, where $S \neq S'$, $\forall i \in S, \forall j \in S'$, $g_i \neq g_j$ or $g_i = g_j = 0$ hold.
(3) If a good does not belong to any set in the division, it must be allocated to the dummy agent 0, i.e., $\forall i$, if $\forall S \in D, i \notin S$ holds, then $g_i = 0$.

Note that we allow the case where some bundle is allocated to the dummy agent 0.

[3] This condition is not contradictory to the second condition, since the second condition involves the union of multiple bundles.

Also, we define *allowed allocations* in level i (denoted as SG_i) as follows.

- SG_i represents the union of all allowed allocations for each element of the division set $SD_i = [D_{i1}, D_{i2}, \ldots]$ of the level i.

To execute the leveled division set protocol (LDS protocol), the auctioneer must pre-define the leveled division set and the reservation prices of goods. Each agent x declares its evaluation value $B(x, S)$ for each bundle S, which may or may not be true. The declared evaluation value of agent x for an allocation G (represented as $v_x(G)$) is defined as $B(x, S)$ if S is allocated to agent x in G, otherwise $v_x(G) = 0$. Also, we define the evaluation value of the dummy agent 0 for an allocation G as the sum of the reservation prices of goods allocated to agent 0.

To search the level in which goods should be sold, the auctioneer calls the procedure LDS(1). LDS(i) is a recursive procedure defined as follows. When the appropriate level is determined, the procedure GVA(i) is called to determine the winners and payments.

Procedure LDS(i)

Step 1: If there exists only one agent $x \in N$ whose evaluation values satisfy the following condition: $\exists D_{ik} \in SD_i, \exists S_x \in D_{ik}$, where $B(x, S_x) \geq R(S_x)$, then compare the results obtained by the procedure GVA(i) defined below and LDS($i+1$), and choose the one[4] that gives the larger utility for agent x. In this case, we say agent x is a *pivotal* agent. When choosing the result of LDS($i+1$), we don't assign any good, nor transfer money, to agents other than x, although the assigned goods for agent x and its payment are calculated as if goods were allocated to the other agents.

Step 2: If there exist at least two agents $x_1, x_2 \in N, x_1 \neq x_2$ whose evaluation values satisfy the following condition: $\exists D_{ik} \in SD_i, \exists D_{il} \in SD_i, \exists S_{x_1} \in D_{ik}, \exists S_{x_2} \in D_{il}$, where $B(x_1, S_{x_1}) \geq R(S_{x_1})$, $B(x_2, S_{x_2}) \geq R(S_{x_2})$, then apply the procedure GVA(i).

Step 3: Otherwise: call LDS($i+1$), or terminate if $i = max\_level$.

Procedure GVA(i)**:** Choose an allocation $G^* \in SG_i$ such that it maximizes $\sum_{y \in N \cup \{0\}} v_y(G)$. The payment of agent x (represented as p_x) is calculated as $\sum_{y \neq x} v_y(G^*_{\sim x}) - \sum_{y \neq x} v_y(G^*)$, where $G^*_{\sim x} \in SG_i$ is the allocation that maximizes the sum of all agents' (including the dummy agent 0) evaluation values except that of agent x.

Note that the procedures in GVA(i) are equivalent to those in the GVA, except that the possible allocations are restricted to SG_i. We say that the *applied level* of the LDS protocol is i if the result of GVA(i) is used.

[4] If the condition of Step 1 is also satisfied for LDS($i+1$), then compare with the results of GVA($i+1$) and LDS($i+2$) also, and so on.

Example 3 *Let us assume there are two goods A and B, the reservation price of each good is 50, the leveled division set is defined as case 1 in Figure 1, and the evaluation values of agents are defined as follows.*

	A	*B*	*AB*
agent 1	*80*	*0*	*110*
agent 2	*0*	*80*	*105*
agent 3	*60*	*0*	*60*

Since there exist two agents whose evaluation values for the set are larger than the sum of the reservation prices (i.e., 100), the condition in Step 2 of $LDS(1)$ *is satisfied; agent 1 obtains both goods by paying 105. Note that this allocation is not Pareto efficient. In the Pareto efficient allocation, agent 1 obtains A and agent 2 obtains B.*

Example 4 *The problem setting is basically equivalent to Example 3, but the evaluation values are defined as follows.*

	A	*B*	*AB*
agent 1	*80*	*0*	*80*
agent 2	*0*	*80*	*80*
agent 3	*60*	*0*	*60*

There exists no agent whose evaluation value of the set is larger than 100. In this case, the condition in Step 3 of $LDS(1)$ *is satisfied, and then the condition in Step 2 of* $LDS(2)$ *is satisfied. As a result, agent 1 obtains A and agent 2 obtains B. Agent 1 pays 60, and agent 2 pays the reservation price 50.*

Example 5 *The problem setting is basically equivalent to Example 3, but the evaluation values are defined as follows.*

	A	*B*	*AB*
agent 1	*80*	*0*	*110*
agent 2	*0*	*80*	*80*
agent 3	*60*	*0*	*60*

There exists only one agent whose evaluation value of the set is larger than 100. The condition in Step 1 of $LDS(1)$ *is satisfied; agent 1 is the pivotal agent. Agent 1 prefers obtaining only A (with the payment 60) to obtaining both A and B (with the payment 100). Therefore, agent 1 obtains A and pays*

60.

Note that in Example 5, B is not allocated to any agent. This might seem wasteful, but this is necessary to guarantee incentive compatibility. In Example 3, if agent 2 declares its evaluation value for the set as 80, the situation becomes identical to Example 5. If we allocate the remaining good B to agent 2, under-bidding becomes profitable for agent 2.

Example 6 *There are three goods A, B, and C. The reservation price for each is 50, and the leveled division set is defined as case 2 in Figure 1. The evaluation values of agents are defined as follows.*

	A	*B*	*C*	*AB*	*BC*	*AC*	*ABC*
agent 1	*60*	*30*	*30*	*90*	*60*	*90*	*120*
agent 2	*30*	*60*	*30*	*90*	*90*	*60*	*120*
agent 3	*30*	*30*	*60*	*60*	*90*	*90*	*120*

The condition in Step 2 of LDS(3) is satisfied. Agents 1, 2, 3 obtain A, B, C, respectively, and each pays the reservation price 50.

4 Proof of Incentive Compatibility

It is obvious that the LDS protocol satisfies individual rationality. We are going to prove that it also satisfies incentive compatibility.

Theorem 1 *The LDS protocol satisfies incentive compatibility even if agents can submit false-name bids.*

To prove Theorem 1, we use the following lemmas.

Lemma 1 *In the LDS protocol, the payment of agent x who obtains a bundle S is larger than (or equal to) the sum of the reservation prices $R(S)$.*

The proof is as follows. Let us assume that the applied level is i. The payment of agent x (represented as p_x) is defined as $p_x = \sum_{y \neq x} v_y(G^*_{\sim x}) - \sum_{y \neq x} v_y(G^*)$. The set of allocations SG_i considered at level i contains an allocation G', which is basically the same as G^*, but all goods in S are allocated to the dummy agent 0 rather than x. The following formula holds.

$$\sum_{y \neq x} v_y(G') = \sum_{y \neq x} v_y(G^*) + R(S)$$

Since $G^*_{\sim x}$ is the allocation that maximizes the sum of all agents' evaluation values (including the dummy agent) except x in SG_i, $\sum_{y \neq x} v_y(G') \leq$

$\sum_{y \neq x} v_y(G^*_{\sim x})$ holds. Thus, the following formula holds.

$$\begin{aligned} p_x &= \sum_{y \neq x} v_y(G^*_{\sim x}) - \sum_{y \neq x} v_y(G^*) \\ &\geq \sum_{y \neq x} v_y(G') - \sum_{y \neq x} v_y(G^*) = R(S) \end{aligned}$$

Lemma 2 *In the LDS protocol, an agent cannot increase its utility by submitting false-name bids.*

The proof is as follows. Let us assume that agent x uses two false names x' and x'' to obtain two bundles $S_{x'}$ and $S_{x''}$, respectively. Also, let us assume that the applied level is i. From Lemma 1, the payments $p_{x'}$ and $p_{x''}$ satisfy $p_{x'} \geq R(S_{x'})$ and $p_{x''} \geq R(S_{x''})$. Now, let us assume that agent x declares the evaluation value $R(S)$ for the bundle $S = S_{x'} \cup S_{x''}$ by using a single identifier. From the condition of a leveled division set, there exists a level $j < i$, where $S \in D_{jl}$, $D_{jl} \in SD_j$ holds. In this case, the condition in Step 1 of LDS(j) is satisfied, i.e., only agent x declares evaluation values that are larger than or equal to the sum of reservation prices. Thus, $\sum_{y \neq x} v_y(G^*_{\sim x}) = R(M)$, and $\sum_{y \neq x} v_y(G^*) = R(M) - R(S)$ hold. As a result, the payment becomes $R(S) = R(S_{x'}) + R(S_{x''}) \leq p_{x'} + p_{x''}$, i.e., the payment of agent x becomes smaller than (or equal to) the case where agent x uses two false names. Similarly, we can show that even when an agent uses more than two identifiers, the payment of such an agent would become smaller (or remain the same) if the agent uses only one identifier.

Lemma 2 states that false-name bids are not effective in the LDS protocol. Now, we are going to show that truth-telling is the dominant strategy for each agent assuming that each agent uses a single identifier.

The following lemma holds.

Lemma 3 *When there exists no false-name bid, and the applied level of the LDS protocol remains the same, an agent can maximize its utility by declaring its true private values.*

The proof is as follows. As long as the applied level is not changed, the possible allocation set SG_i is not changed. The payment of agent x is defined as $\sum_{y \neq x} v_y(G^*_{\sim x}) - \sum_{y \neq x} v_y(G^*)$. Let us represent the true private value of agent x of an allocation G as $u_x(G)$. The utility of agent x is represented as $u_x(G^*) + \sum_{y \neq x} v_y(G^*) - \sum_{y \neq x} v_y(G^*_{\sim x})$, i.e., the difference between the evaluation value and the payment. The third item of this formula is determined independently of agent x's declaration if there exists no false-name bid. Therefore, agent x can maximize its utility by maximizing the sum of the first two items. On the other hand, the allocation G^* is chosen so that

$\sum_{y \in N \cup \{0\}} v_y(G) = v_x(G) + \sum_{y \neq x} v_y(G)$ is maximized. Therefore, agent x can maximize its utility by declaring $v_x(G) = u_x(G)$, i.e., declaring its true private values.

Next, we show that an agent cannot increase its utility by changing the applied level.

Lemma 4 *An agent cannot increase its utility by over-bidding so that the applied level decreases.*

The proof is as follows. Let us assume that when agent x truthfully declares its utility, the applied level is i, and by over-bidding, the applied level is changed to $j < i$. In that case, for each bundle S included in the divisions of level j, agent x's evaluation value of S must be smaller than the sum of the reservation prices $R(S)$; otherwise, level j is applied when agent x tells the truth. On the other hand, by Lemma 1, the payment for a bundle S is always larger than the sum of the reservation prices $R(S)$, which means that agent x cannot obtain a positive utility by over-bidding.

Lemma 5 *An agent cannot increase its utility by under-bidding so that the applied level increases.*

The proof is as follows. Agent x can increase the applied level only in the following two cases.

(1) Agent x is the pivotal agent when agent x truthfully declares its evaluation values.
(2) By under-bidding, another agent y becomes the pivotal agent.

In the first case, let us assume if agent x tells the truth, agent x becomes the pivotal agent at level i. Also, let us assume the applied level becomes $j > i$ when agent x does under-bidding. If agent x prefers the result of level j, then the result of LDS(j) must be chosen in the procedure of Step 1 when agent x tells the truth. Therefore, under-bidding is useless. In the second case, agents other than y cannot obtain any goods; the utility of agent x becomes 0. In both cases, agent x cannot increase its utility by under-bidding.

From these lemmas, we can derive Theorem 1. □

When proving Lemma 2, we use the second condition of the leveled division set, i.e., a union of multiple bundles must appear in an earlier level. In other words, the leveled division set is constructed so that an agent who is willing to buy larger bundles has a higher priority. Therefore, submitting false-name bids and buying goods separately does not make sense at all.

5 Evaluation

In the LDS protocol, we can expect that the social surplus and the revenue of the seller can vary significantly according to the leveled division set and reservation prices. In this section, we show how the social surplus changes according to the reservation prices using a simulation in a simple setting where there are only two goods A and B, and the leveled division set is defined as case 1 in Figure 1.

We determine the evaluation values of agent x by the following method.

- Determine whether the goods are substitutional or complementary for agent x, i.e., with probability p, the goods are substitutional, and with probability $1 - p$, the goods are complementary.
  - When the goods are substitutional: for each good, randomly choose its evaluation value from within the range of $[0, 1]$. The evaluation value of the set is the maximum of the evaluation value of A and that of B (having only one good is enough).
  - When the goods are complementary: the evaluation value of A or B is 0. Randomly choose the evaluation value of the set from within the range of $[0, 2]$ (all-or-nothing).

Figure 2 shows the result where $p = 0.5$ and the number of agents N is 10. We created 100 different problem instances and show the average of the social surplus by varying the reservation price. Both A and B have the same reservation price. For comparison, we show the social surplus of the GVA (assuming there exists no false-name bid), i.e., the Pareto efficient social surplus [5], and the social surplus of the set protocol.

Figure 3 shows the result where $p = 0.7$.

When the reservation price is small, the results of the LDS protocol are identical to the set protocol. We can see that by setting an appropriate reservation price the obtained social surplus becomes larger than that for the set protocol. When the probability that the goods are substitutional becomes large, the difference between the set protocol and the GVA, and the difference between the set protocol and the LDS protocol also become large.

[5] We cannot tell the result of the GVA when agents can submit false-name bids, since there exists no dominant strategy equilibrium.

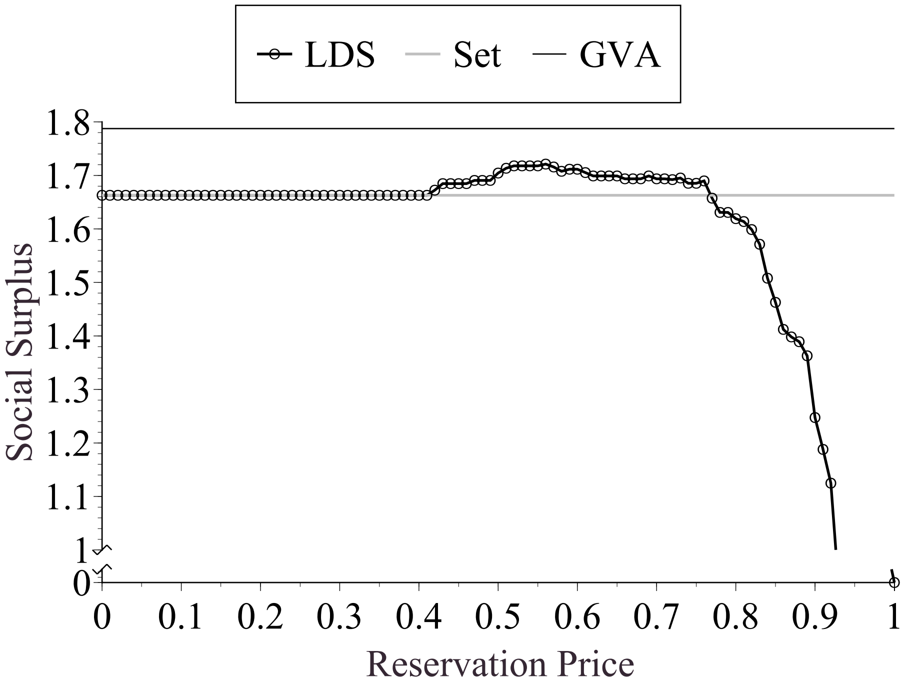

Fig. 2. Comparison of Social Surplus ($p = 0.5$)

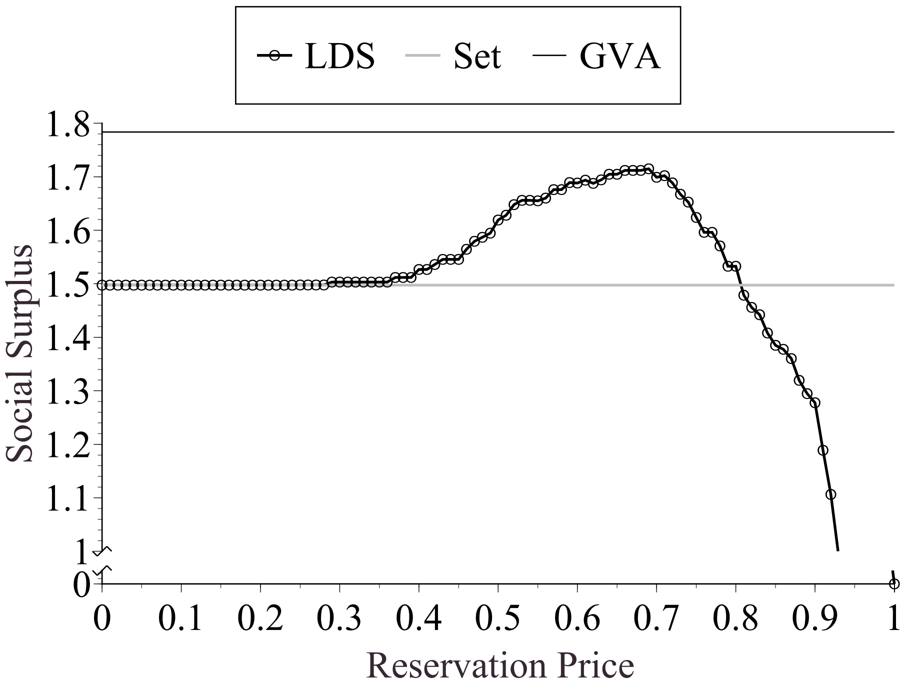

Fig. 3. Comparison of Social Surplus ($p = 0.7$)

6 Discussions

As far as the authors know, the LDS protocol is the first non-trivial protocol that is robust against false-name bids. One shortcoming of this protocol is that when the leveled division set and reservation prices are not determined appropriately, there is a chance that some goods cannot be sold. In that case, the social surplus and the revenue of the seller might be smaller than that for the set protocol.

One advantage of the LDS protocol over the GVA is that it requires less communication/computation costs. To execute the GVA, the bidder must declare its evaluation values for all possible bundles. Also, the seller must solve a complicated optimization problem to determine the winners and their payments [2,8,11]. In the LDS protocol, the allowed bundles are pre-determined, and bidders need to submit bids only for these bundles. Furthermore, the search space of the possible allocations is much smaller than the search space that must be considered in the GVA [6]. In [8], the idea of restricting the possible bundles where agents can submit bids is presented to efficiently determine winners in combinatorial auctions. The leveled division set introduced in this paper is intended to guarantee robustness against false-name bids, and there is no direct relationship between the leveled division set and the methods of dividing goods described in [8].

As mentioned in Section 1, we can consider false-name bids as a very restricted subclass of collusion and there exist some over-lapping issues. In the GVA, agents can collude to decrease their payments. For example, in Example 1, agent 1 and agent 3 can collude to decrease their payments by increasing their bids. Increasing its own bid does not decrease its own payment, but can decrease the payment of the other agent. The LDS protocol can be a safeguard against such collusion, since if the reservation prices are small, the goods are sold to agent 2 anyway, and if the reservation prices are high, each agent needs to pay the reservation price of each good.

7 Conclusions

In this paper, we presented a new combinatorial auction protocol (LDS protocol) that is robust against false-name bids. This protocol satisfies individual rationality and incentive compatibility and can achieve a relatively good, but not Pareto efficient, social surplus. The main ideas of the LDS protocol are to restrict the possible divisions of goods by a leveled division set and to utilize the reservation prices of goods to determine the applied level. Simulation results showed that this protocol can achieve a better social surplus than that for the set protocol.

[6] Of course, we cannot achieve a Pareto efficient allocation since we restrict the possible allocations.

One remaining research issue is how to determine the leveled division set and reservation prices that maximize the social surplus or the revenue of the seller. We are working on a method to determine the appropriate leveled division set and reservation prices based on certain expectations of bidders' evaluation values, and a method to update this expectations based on historical information of previous auctions.

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